



ASSESSING THE SEPARATION DISTANCE BETWEEN THE GEOID AND THE REFERENCE ELLIPSOID IN PART OF JALINGO, TARABA STATE

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ABSTRACT

This research is aimed at assessing the separation distance between the Geoid and the reference Ellipsoid in selected points within Taraba State University Jalingo, Nigeria. GPS instrument was used to observe the coordinates and heights of these points in differential mode. Thereafter the separation distance between the geoid and the reference ellipsoid (geoid undulation) values were computed with earth gravity models 2008 (EGM2008). The EGM2008 uses geopotential coefficients and spherical harmonic analysis to determine the geoid undulation. The results obtained were plotted and the configuration of the local geoid was obtained. Orthometric heights were subsequently computed with reference to geoid from the earth's surface. This is the height preferred by users because of its relationship with the Mean Sea Level which is approximated to the geoid. The computation of orthometric height was aided by the use of Microsoft Excel. The height difference between the Geoid and the reference Ellipsoid base on calculated root mean square error 18.246m shows that there is a significant and adequate distance between the Geoid and the reference Ellipsoid. It was clearly shown that the geoid is an equipotential surface and ellipsoidal and orthometric heights also follow the same pattern in that area. Furthermore, the Contour Map along with the DTM of the study area was plotted, the trends of the plots of orthometric and Ellipsoidal heights followed the same pattern. This is an indication that the two height systems are true representation of the same terrain.

Key words: Ellipsoid, Ellipsoidal Height, Geoid, Geoidal Undulation and Orthometric Height.

1. INTRODUCTION

There are interesting relationships between the topographical surface of the earth (Geoid) and its figures of approximations (Ellipsoid). This relationship is clearly seen by their height systems: That is, height of a point on the earth's surface measured along the ellipsoidal normal to the surface of the ellipsoid (ellipsoidal height 'h') and the height of a point on the earth's surface measured along the plumb line, normal to the geoid, (orthometric height 'H'). Orthometric height coincides with the direction of gravity vector which is at all points normal to the surface of the geoid. Orthometric height at every point therefore is a function of gravity at that point. The surface of the geoid is higher than the reference ellipsoid wherever there is a positive gravity anomaly and lower than the reference ellipsoid wherever there is a negative gravity anomaly. The difference between these two height systems (ellipsoidal and orthometric heights) is the geoidal undulation variations. The geoid undulation determination



in the study area was done by global positioning system (GPS) observations on differential mode.

In the recent past, one interesting and challenging tasks in the field of geodetic surveying is the accurate determination of orthometric heights from GNSS, in particular GPS measurements for a local geodetic datum (Al-Ghamdi and Dawod, 2013, Lee et al., 2012). This poses a challenge for high determination in engineering works such as engineering surveys or 3D coordinate transformation and mapping (Featherstone et al., 2001; Fotopoulos, 2003). Converting the GPS height to a physical meaning require the determination of the separation distance between the geoid and the reference ellipsoid (geoid undulation). The EGM96 and EGM08 are some of the models used to calculate the geoid undulation in order to determine the orthometric height from GPS measurements (Do, 2011) and in Heiskanen and Moritz, (1967), the Stokes' integral technique is used to compute the geoid undulation.

EGM08 model is mostly used for height conversion in some countries with a relatively high degree of accuracy. The EGM08 is good enough for geodetic applications like determining the topographic heights of points on the globe that require the geoid which approximates Mean Sea Level (MSL) as the datum/reference surface (Yilmaz et al., 2010; Abeho et al., 2014). The EGM08 was preceded by EGM96 which had a lower degree of accuracy (Pavlis et al., 2008). EGM08 is capable of obtaining a sufficiently accurate model of the gravity field over the surface of the earth (Kotsakis et al., 2009; Kotsakis and Sideris, 1999). Dawod, (2008)); Dawod et al., (2010) and Soycan, (2014) observed that, EGM08 derived geoid heights can reach the accuracy of regional or local geoid models after modeling the differences between the GPS/leveling geoid heights and EGM08 derived geoid heights at identified control points. It is therefore the intent of this research to assess the separation distance between geoid and reference ellipsoid of Taraba State University, which would be useful in modeling the geoid undulation.

1.1 Statement of the Problem

In geodesy, the figure of the earth is represented by the surface of the geoid and the geoid experiences deformation from time to time due to artificial and natural occurrences. The determination of the figure of the earth with time has remained a major task in the world of geodesy. The shifting of masses from one point to the other on the earth's surface result to the variations in the earth's gravity values across the same points. The separation distance between the geoid and the reference ellipsoid at a point, also known as geoid undulation is a function of gravity at that point, the determination of geoid undulation in the study area with the use of earth gravity model in seeking a more convenient way of finding orthometric heights rather than through spirit leveling constitute some problems that need to be addressed. This research aimed to address this problem by assessing the separation distance between the geoid and the reference ellipsoid with a view to ascertain the behavior of the equipotential



surface of the geoid for a reliable height referencing.

1.2 Justification of the Research

Separation between the geoid and the reference ellipsoid shows the geoidal configurations of the study area which is a significant aspect of geophysical study and the analysis of geological structures of the area separation. The rigorous determination of orthometric heights by conventional method of spirit leveling has been removed by the use of geoid height calculators and other related software which uses ellipsoidal height, longitude and latitude determined by GPS observations to compute the between the geoid and the reference ellipsoid of the points and in turn the orthometric heights of the respective points selected in the study area. The results from this study serve as an alternative approach to the study of the nature of geoid and the ellipsoid as reference datum for heights in survey measurements.

1.3 Scope of the Research

The scope of this research is limited to the determination of ellipsoidal heights (h), longitude (λ) and Latitude (ϕ) using DGPS observations. Determination of separation distance between the geoid and the reference ellipsoid also known as the geoidal undulation (N), using EGMs. Determination of orthometric heights (H) from h and N using geoid height calculator, EGM2008. Plotting of different sets of undulation values computed from earth gravity models to see the nature and behavior of the geoid in that area. Assessing the results to see the relationship between the values of undulation systems in the selected points of the study area.

1.4 Study Area

Jalingo is the capital of Taraba state, the state is located on latitudes $8^{\circ} 50'N$ and $09^{\circ} 10'N$ of the equator and longitude $10^{\circ} 8' E$ and $11^{\circ} 50' E$ of the greenwich meridian. There are 16 local government areas that make up the state. The state shares boundaries with Bauchi and Gombe states in the north, Adamawa state in the east, and the Cameroon republic in the south. The state is bounded along its western side by Plateau, Nassarawa and Benue states. The state has a land area of 60,291km² with a population of about 2.5 million as projected from 2006 national population census. The state is divided into three senatorial districts (Taraba north, central and south). Taraba State is regarded as nature's gift to the nation because of its abundant natural resource endowment.

The state is well endowed with abundant solid mineral resources, surface water resources, arable and grazing land. The major occupation of the people of Taraba State is agriculture. The state is blessed with climate and vegetation types that cut across the country, ranging from a more humid climate and forest vegetation in the south to a more seasonal wet and dry climate and savanna vegetation in the north. These favour the growth of tree crops such as palm oil, banana/plantain, orange etc. Root crops in the state include cassava, potato and yam, while cereals include maize, rice, millet, sorghum and guinea corn. Cash crops produced



in the state include coffee, tea and groundnuts (Oruonye and Abbas, 2011).

In addition, cattle, sheep and goats are reared in large numbers, especially on the Mambilla Plateau, and along the Benue and Taraba river valleys (Oruonye and Abbas, 2011). Communities living on the banks of River Benue, River Taraba, River Donga and Ibi (on the bank of River Benue) engage in fishing activities all year round. The state is also a tourist haven in the country. The famous Mambilla plateau with its beautiful landscape characterized by valleys and waterfalls and its lush green vegetation makes the state a potential pace-setter in the field of tourism in the country. The Gashaka-Gumti National Park located at the foot of the Mambilla plateau is another major outstanding tourist landmark.

Taraba state university is the study point; it is located in Jalingo, it is bounded to the north by Lau local government Area, to the east by Yorro local government Area, to the south and west by Ardo Kola Local government area (figure1). The main campus of the University is located along Gembu road at Advance Teachers College (A.T.C), beside college of Agriculture Jalinjo Taraba state. The institution was established in 2008 during governor Danbaba Danfulani Suntai regime.

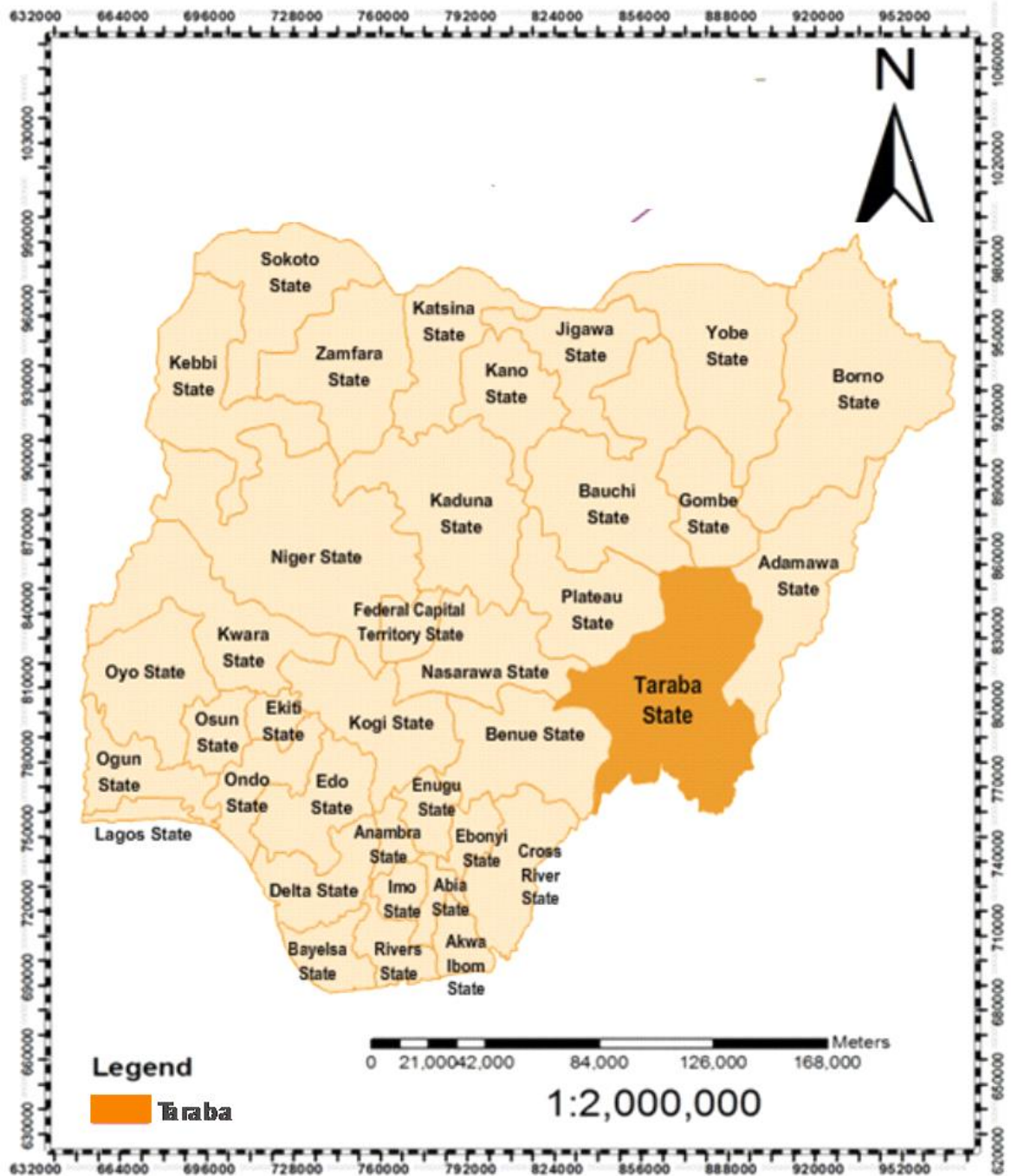


Figure 1: Map of Nigeria Showing Taraba State

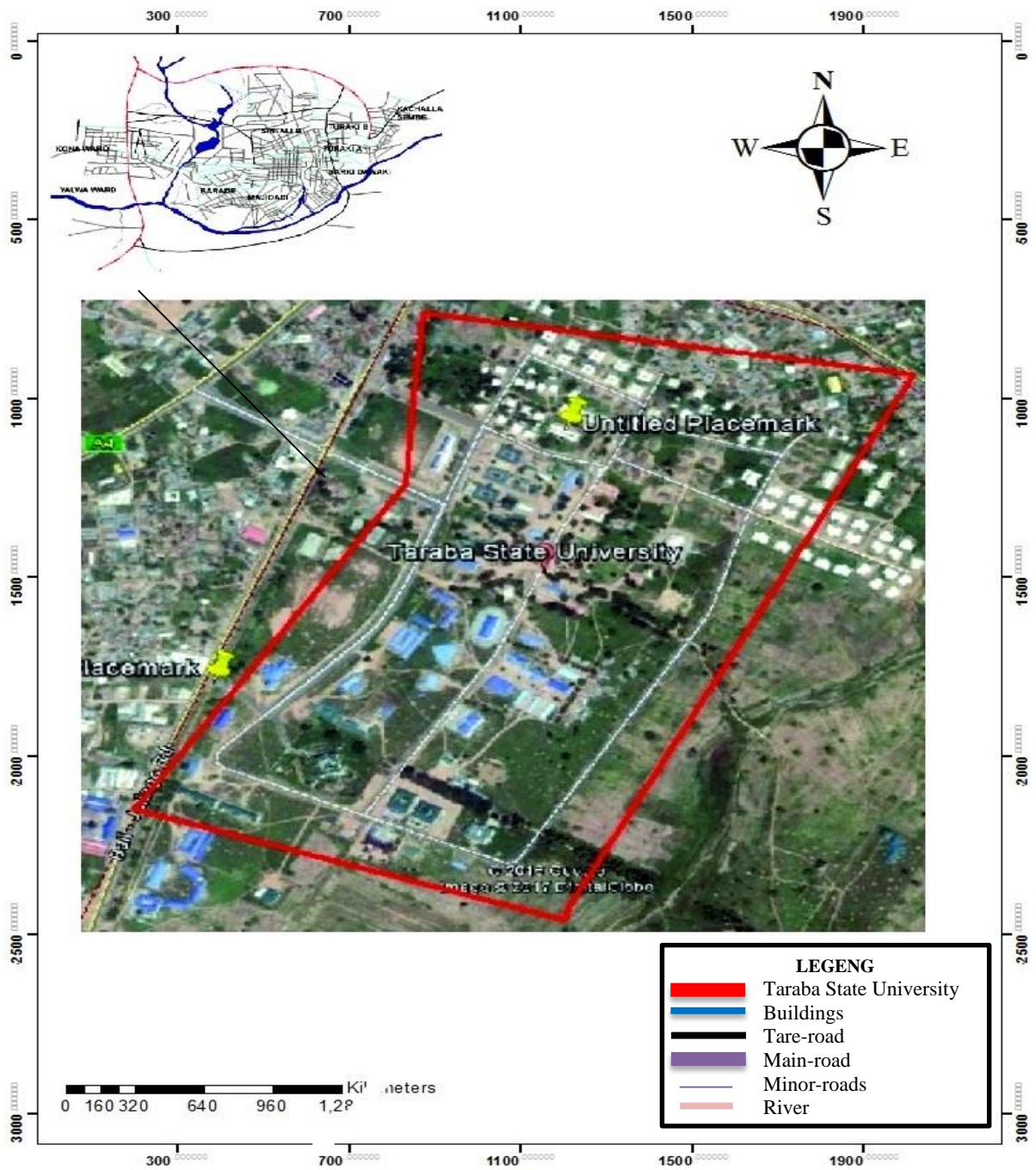




Figure 2: Taraba State University Image

2. MATERIALS AND METHOD

The material and equipments used includes the following:

- a. GPS topcone FC 20 receiver and its accessories
- b. A Lab top computer fitted with GNSS Solution Software, Geoid Height Calculators and Statistical calculators

2.1 Methodology

The method adopted in this research involves collection of data for Northing's, Easting's and Ellipsoidal heights and processing of the data to obtain Geoid height. The Geoid heights were subsequently used in the computation of Orthometric heights for each point. A map was produced to show the pictorial representation of Geoid height, Ellipsoid height and Orthometric heights of the area.

2.2 Data Acquisition

The data utilized for the research includes latitude (Φ), longitude (λ) and ellipsoidal heights (h) of points obtained using differential (GPS) receivers. These set of data were obtained from the site by means of direct field observation. Differential GPS was used to collect data both at the base and rover position in the field for GPS observation. The control point obtained during the office reconnaissance was first located and confirmed with the use of DGPS receiver, after the confirmation, the base station was set right on it and the base station receiver was initialized with the rover receiver, then the rover receiver was moved (stop-and-go) randomly within the study area for collection of data in real time.

2.3 Reconnaissance Survey

The basic principle of surveying requires that reconnaissance be done before proceeding on the actual field work in any survey job. In this research work, reconnaissance was carried out and the following factors were considered namely: the nature of the terrain, sky visibility in GPS observation, the suitability of the station and the method to be adopted, as well as the general information available about the task to be carried out

2.4 Procedure for Surveying with Differential GPS (Topcone FC 20)

The Differential Global Positioning System (DGPS) topcone FC 20 receiver was used in static survey mode to capture data over Twenty-seven control points. The Master receiver was setup on a known control point (CSTT1047) where temporary adjustment was done. The receiver was turned on and satellites were acquired. The master was calibrated to a static mode then initialized. After configuring the master, the rover was also turned on to the survey mode and calibrated on the same point (CSTT1047). The difference between the master and the rover observation was seen to be negligible. The rover was then moved to TBM1 and it was allow to occupy the point automatically at 100% initialization and the



same data acquisition procedure was adopted and repeated for all the remaining twenty-six points within the study area.

2.5 Data Quality

Quality of data used for any experiment can be determined by the validity and reliability of such data based on the assumption that the observers of such data are trust worthy and experienced (Idowu, 2005). The validity of the data is measured by the precision of the instrument used while the reliability of data is determined by the accuracy of such data. The instrument used was tested and found to be fit for use. The test was done by repeated measurements of coordinated points around the study area. The result obtained showed a high degree of closeness of the data which proves its validity. Subsequently the result was also compared with values of the coordinated points. It was discovered that the one measured with the instrument used in this research was satisfactorily close to the known values. This shows that the two results agreed with each other proving its reliability. Based on these facts it can be said that the data used are valid and reliable, and therefore the quality of the data that was used in this work can be said to be trustworthy and of good quality.

2.6 Data Processing

The data (Table 1) acquired with DGPS was in universal traverse mercator (U.T.M) which was converted to Decimal Degree Minute and Seconds using Microsoft Excel (Table 2).



Table1: Coordinates and Height of points

| POINT ID | NORTHING | EASTING | ELLIPSOIDAL HEIGHT |
|----------|-------------|-------------|--------------------|
| CSTT1047 | 754665.190 | 985659.644 | 221.045 |
| TBM1 | 754843.180, | 976011.493 | 215.961 |
| TBM2 | 754610.492, | 985162.667 | 232.627 |
| TBM3 | 754692.082, | 985116.939 | 233.109 |
| TBM4 | 754760.593, | 985080.594, | 234.314 |
| TBM5 | 754834.123, | 985038.952, | 235.531 |
| TBM6 | 755106.486, | 984935.041, | 235.179 |
| TBM7 | 755215.586, | 984881.754, | 232.469 |
| TBM8 | 755338.790, | 984741.522, | 229.294 |
| TBM9 | 755521.169, | 984591.397, | 223.270 |
| TBM10 | 755524.042, | 984583.322, | 220.767 |
| TBM11 | 755523.283, | 984578.288, | 223.301 |
| TBM12 | 755559.883, | 984498.591, | 226.158 |
| TBM13 | 755561.781, | 984498.416, | 225.214 |
| TBM14 | 755563.553, | 984496.762, | 226.085 |
| TBM15 | 755767.839, | 984402.467 | 227.032 |
| TBM16 | 756054.860, | 984346.232, | 220.412 |
| TBM17 | 756032.993, | 984336.584, | 217.446 |
| TBM18 | 755587.735, | 984051.477, | 235.171 |
| TBM19 | 755405.259, | 983881.035, | 240.232 |
| TBM20 | 755236.926, | 983964.066, | 238.624 |
| TBM21 | 755107.392, | 984040.363, | 234.716 |
| TBM22 | 755074.447, | 984055.904, | 232.034 |
| TBM23 | 755063.239, | 984062.899, | 233.774 |
| TBM24 | 754843.007, | 984191.625, | 240.336 |
| TBM25 | 754422.060, | 984432.977, | 248.043 |
| TBM26 | 754049.605, | 983888.587, | 258.921 |
| TBM27 | 754701.678, | 985416.748, | 227.259 |



Table 2: Coordinates in Decimal Degree

| Pt. ID | LATITUDE | | | LONGITUDE | | | LATITUDE | LONGITUDE |
|----------|----------|----|----|-----------|----|----|-------------|-------------|
| | D | M | S | D | M | S | Dec. Degree | Dec. Degree |
| CSTT1047 | 08 | 54 | 35 | 11 | 18 | 58 | 8.90966 | 11.31600 |
| TBM1 | 08 | 49 | 21 | 11 | 19 | 01 | 8.82245 | 11.31707 |
| TBM2 | 08 | 54 | 19 | 11 | 18 | 56 | 8.90517 | 11.31547 |
| TBM3 | 08 | 54 | 17 | 11 | 18 | 58 | 8.90475 | 11.31621 |
| TBM4 | 08 | 54 | 16 | 11 | 19 | 01 | 8.90442 | 11.31683 |
| TBM5 | 08 | 54 | 15 | 11 | 19 | 03 | 8.90404 | 11.31750 |
| TBM6 | 08 | 54 | 11 | 11 | 19 | 12 | 8.90309 | 11.31997 |
| TBM7 | 08 | 54 | 09 | 11 | 19 | 15 | 8.90260 | 11.32096 |
| TBM8 | 08 | 54 | 05 | 11 | 19 | 19 | 8.90132 | 11.32207 |
| TBM9 | 08 | 53 | 60 | 11 | 19 | 25 | 8.89996 | 11.32372 |
| TBM10 | 08 | 53 | 60 | 11 | 19 | 25 | 8.89988 | 11.32374 |
| TBM11 | 08 | 53 | 59 | 11 | 19 | 25 | 8.89984 | 11.32374 |
| TBM12 | 08 | 53 | 57 | 11 | 19 | 27 | 8.89912 | 11.32407 |
| TBM13 | 08 | 53 | 57 | 11 | 19 | 27 | 8.89911 | 11.32408 |
| TBM14 | 08 | 53 | 57 | 11 | 19 | 27 | 8.89910 | 11.32410 |
| TBM15 | 08 | 53 | 54 | 11 | 19 | 33 | 8.89824 | 11.32595 |
| TBM16 | 08 | 53 | 52 | 11 | 19 | 43 | 8.89771 | 11.32855 |
| TBM17 | 08 | 53 | 51 | 11 | 19 | 42 | 8.89762 | 11.32836 |
| TBM18 | 08 | 53 | 42 | 11 | 19 | 27 | 8.89507 | 11.32429 |
| TBM19 | 08 | 53 | 37 | 11 | 19 | 21 | 8.89354 | 11.32262 |
| TBM20 | 08 | 53 | 39 | 11 | 19 | 16 | 8.89430 | 11.32110 |
| TBM21 | 08 | 53 | 42 | 11 | 19 | 12 | 8.89500 | 11.31993 |
| TBM22 | 08 | 53 | 43 | 11 | 19 | 11 | 8.89514 | 11.31963 |
| TBM23 | 08 | 53 | 43 | 11 | 19 | 10 | 8.89521 | 11.31953 |
| TBM24 | 08 | 53 | 47 | 11 | 19 | 03 | 8.89638 | 11.31753 |
| TBM25 | 08 | 53 | 55 | 11 | 18 | 49 | 8.89859 | 11.31372 |
| TBM26 | 08 | 53 | 37 | 11 | 18 | 37 | 8.89369 | 11.31031 |
| TBM27 | 08 | 54 | 27 | 11 | 18 | 59 | 8.90746 | 11.31632 |

2.7 Global Geopotential Models (GGMs)



The acquired data was used to calculate the geoidal height using EGM2008 with the mathematical model derived from Global Geopotential models (GGMs). GGMs describe the Earth's gravitational potential in terms of an infinite series of spherical harmonics outside the Earth attracting masses (equation1). They are determined by a combination of satellite and terrestrial observations and used as reference fields in the determination of local and regional geoids. The geopotential is usually given as a truncated set of harmonic coefficients, obtained when solving a Laplace equation in spherical coordinates described (Tapley ,*et al* 2005).

$$V(r, \theta, \lambda) = \frac{GM}{r} \left\{ 1 + \sum_{n=1}^{N_{max}} \sum_{m=2}^1 \left(\frac{a}{r}\right)^n [\bar{C}_{nm} \cos m\lambda + \bar{S}_{nm} \sin m\lambda] \bar{P}_{nm}(\sin \theta) \right\} \quad (1)$$

Where:

| | |
|------------------------------|---|
| GM | Earth's gravity constant |
| r | magnitude of radius vector |
| n, m | degree and order of spherical harmonics |
| \bar{P}_{nm} | Legendre functions |
| $\bar{C}_{nm}, \bar{S}_{nm}$ | Coefficients of spherical harmonics |
| θ | Latitude |
| λ | Longitude |
| v | Disturbing potential |

The disturbing potential T at a point $V(r, \theta, \lambda)$ is the differences between the actual gravity potential of the Earth and the normal potential associated with the a rotating equipotential ellipsoid at V . Based on equation 1 the spherical harmonic representation of T is :

$$T(r, \theta, \lambda) = \frac{GM}{r} \left\{ 1 + \sum_{n=1}^{N_{max}} \sum_{m=2}^1 \left(\frac{a}{r}\right)^n [\bar{C}_{nm} \cos m\lambda + \bar{S}_{nm} \sin m\lambda] \bar{P}_{nm}(\sin \theta) \right\} \quad (2)$$

Equation 2 was expanded for several numerous processes to get the element of the Earth's gravity field such as gravity anomalies (Δg) and geoid height (N) in equation 3 and 4 respectively. The relationship between the coefficient of spherical harmonic with gravity anomalies (Δg_{GM}) and geoidal height (N_{GM}) is given by the following formula, respectively

$$\Delta g_{GM} = \frac{GM}{r^2} \left\{ 1 + \sum_{n=2}^l (n-1) \sum_{m=0}^l \left(\frac{a}{r}\right)^n [\bar{C}_{nm} + \bar{S}_{nm} \sin m\lambda] \bar{P}_{nm}(\sin \theta) \right\} \quad (3)$$

$$N_{GM} = \frac{GM}{r\gamma} \left\{ 1 + \sum_{n=2}^l (n-1) \sum_{m=0}^l \left(\frac{a}{r}\right)^n [\bar{C}_{nm} + \bar{S}_{nm} \sin m\lambda] \bar{P}_{nm}(\sin \theta) \right\} \quad (4)$$

After calculating the geoidal height from EGM2008 the height value was used to compute the Orthometric height in Microsoft Excel using the following model

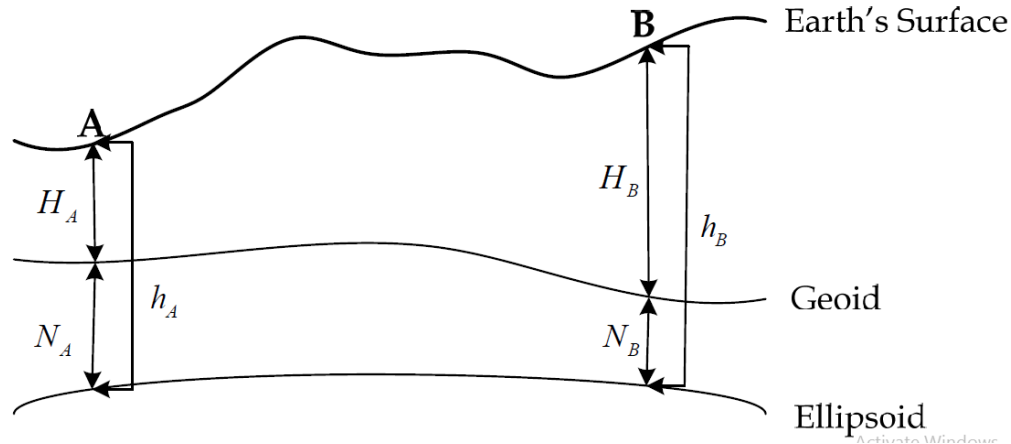


Figure 3: Geoid, Ellipsoid and Earth Surface

$$H = h - N \quad (5)$$

Where

N = Geoidal height or Geoidal undulation

h = Ellipsoidal height with respect to a reference ellipsoid

H = Orthometric height based on geoid

In order to evaluate the adequacy of the EGM2008, root mean square error (RMSE) was employed. The mean difference N_{mean} is the average of the geoidal height differences, N_j for EGM2008 model. The mean was computed using equation 6

$$N_{mean} = \frac{1}{n} \sum_{j=1}^n N_j \quad (6)$$

Where $j = 1, 2, 3, \dots, n$ and $I = 1, 2, 3, \dots, n$.

The root mean square (RMSE) value was computed using equation 7

$$RMSE = \sqrt{\frac{\sum_{j=1}^n N_j^2}{n}} \quad (7)$$

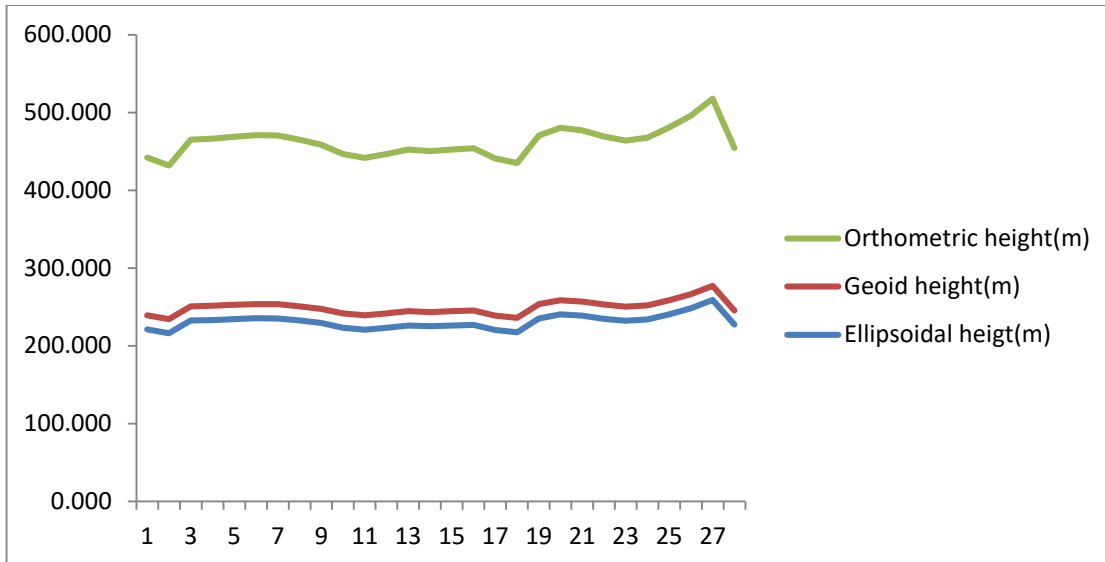
3. PRESENTATION OF RESULTS

The result includes: final computed geoidal undulation (separation between reference ellipsoid and the geoid) and orthometric heights of the selected points at the study area (Table 3). Figure 4a and b shows a chart profile depicting ellipsoid heights, geoidal heights (separation between reference ellipsoid and the geoid) and orthometric heights of the points in the study area. Figure 5a, b and c are the contour map and digital terrain Model (DTM) of the study area derived by using ellipsoidal height, orthometric height and geoidal height of the study area.

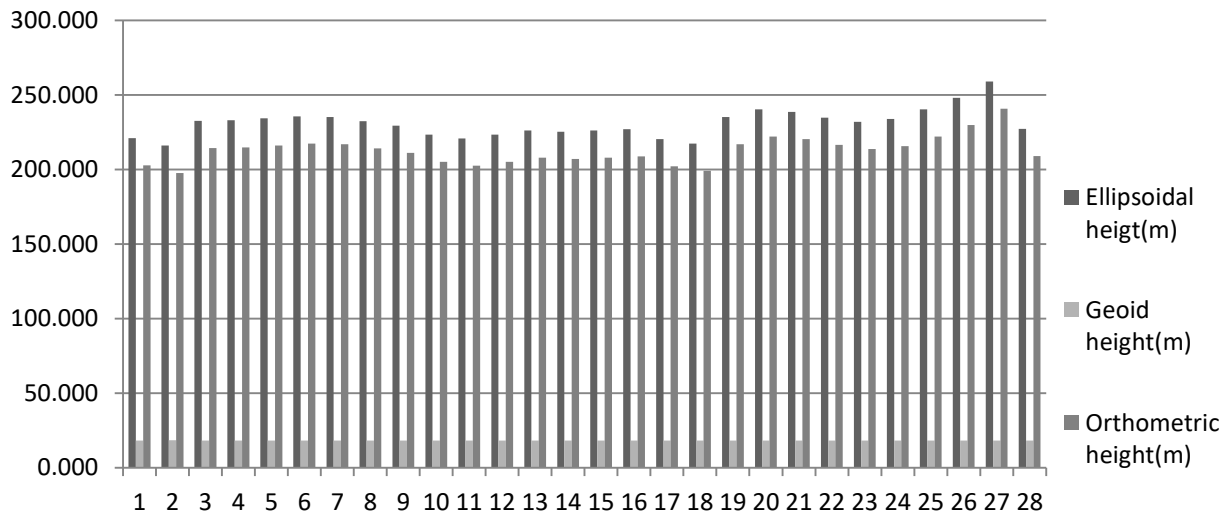


Table 3: Geoidal Undulation and Orthometric Height

| Station | Ellipsoidal height(m) | Geoidal Undulation (m) (Geoidal Height) | Orthometric height(m) |
|----------|-----------------------|---|-----------------------|
| CSTT1047 | 221.045 | 18.240 | 202.805 |
| TBM1 | 215.961 | 18.274 | 197.687 |
| TBM2 | 232.627 | 18.241 | 214.386 |
| TBM3 | 233.109 | 18.242 | 214.867 |
| TBM4 | 234.314 | 18.242 | 216.072 |
| TBM5 | 235.531 | 18.242 | 217.289 |
| TBM6 | 235.179 | 18.243 | 216.936 |
| TBM7 | 232.469 | 18.244 | 214.225 |
| TBM8 | 229.294 | 18.245 | 211.049 |
| TBM9 | 223.270 | 18.246 | 205.024 |
| TBM10 | 220.767 | 18.246 | 202.521 |
| TBM11 | 223.301 | 18.246 | 205.055 |
| TBM12 | 226.158 | 18.246 | 207.912 |
| TBM13 | 225.214 | 18.246 | 206.968 |
| TBM14 | 226.085 | 18.246 | 207.839 |
| TBM15 | 227.032 | 18.247 | 208.785 |
| TBM16 | 220.412 | 18.248 | 202.164 |
| TBM17 | 217.446 | 18.248 | 199.198 |
| TBM18 | 235.171 | 18.248 | 216.923 |
| TBM19 | 240.232 | 18.248 | 221.984 |
| TBM20 | 238.624 | 18.247 | 220.377 |
| TBM21 | 234.716 | 18.247 | 216.469 |
| TBM22 | 232.034 | 18.247 | 213.787 |
| TBM23 | 233.774 | 18.247 | 215.527 |
| TBM24 | 240.336 | 18.246 | 222.09 |
| TBM25 | 248.043 | 18.244 | 229.799 |
| TBM26 | 258.921 | 18.245 | 240.676 |
| TBM27 | 227.259 | 18.241 | 209.018 |



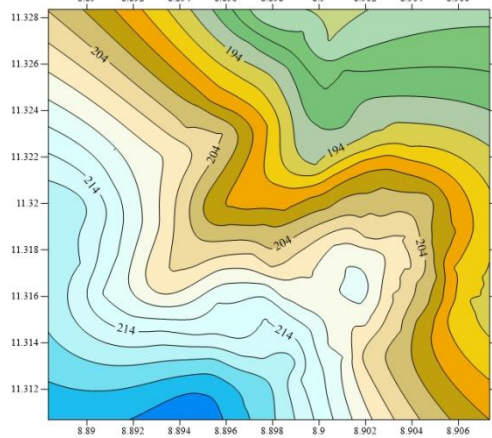
a: Orthometric, Geoidal and Ellipsoidal Height profile



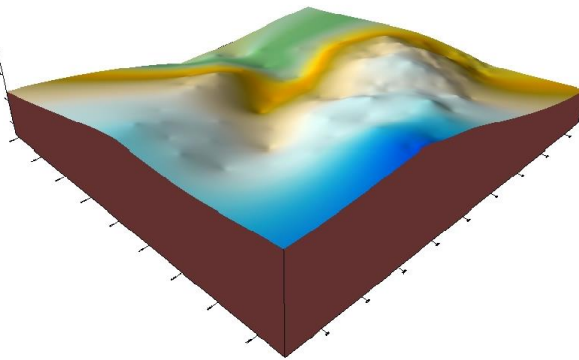
b: Bar Chart Showing Orthometric, Geoidal and Ellipsoidal Height profile

Figure 4: Chart of Heights

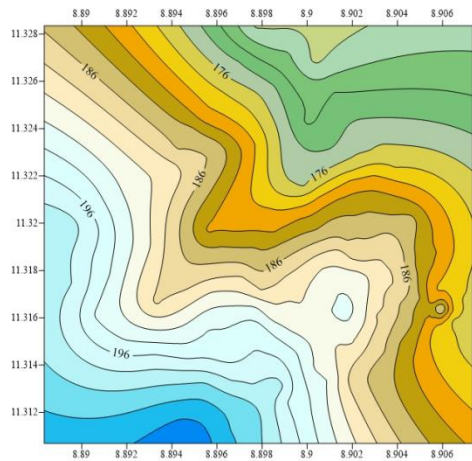




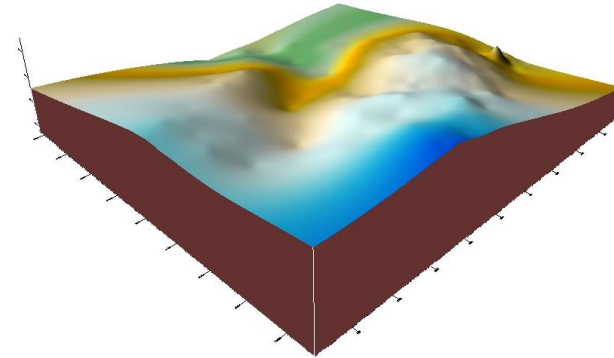
a: Contour Line using Ellipsoidal Height



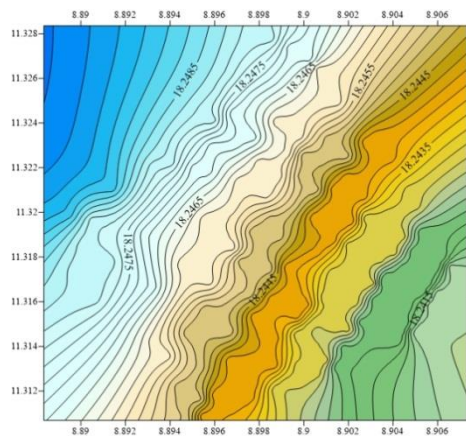
D T M Using Ellipsoidal Height



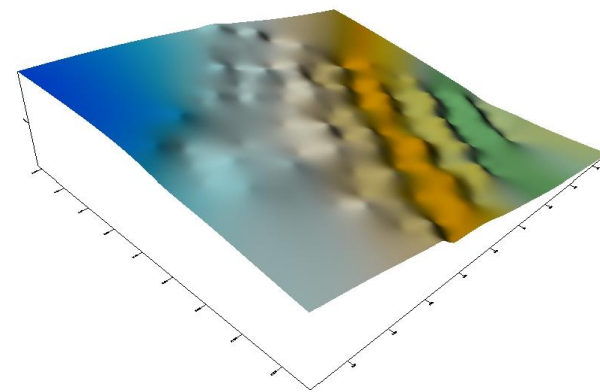
b: Contour Line Using Orthometric Height



D T M Using Orthometric Height



c: Contour Line Using Geoidal Height



D T M Using Geoidal Height

Figure 5: Contour lines and DTM



4. DISCUSSION OF RESULTS

The result presented includes the ellipsoidal height, orthometric height and the separation distance between the geoid and the reference ellipsoid (geoidal undulation). The geoidal undulation values (table 3) are all positive and this is an indication that the geoid at all points of the study area is above the ellipsoid. The values of geoidal undulation could be as a result of two possible factors: The geoid is affected by the distribution of mass of land above mean sea level and the elevated area whose center of gravity is outside the ellipsoid causes an upward attraction leading to local elevation of the geoid above the ellipsoid. The other factor is that the excess mass under the ellipsoid forces the geoid to bend upward which gives positive geoid undulation over mass excess under the ellipsoid. There is a relatively flat topography and a heterogeneous mass distribution due to changes in development and this has affected the geoidal undulation in the area. There is also a significant and adequate distance between the geoid and the reference ellipsoid base on the calculated root mean square error of 18.246m (equation 7).

Furthermore, Figure 4 indicated that the geoid is an equipotential surface considering the trends of the plotted ellipsoidal height, orthometric height and geoidal undulation. To further buttress equipotentiality of the geoid, the contour map along with the DTM of the study area plotted (figure 5) seem to also follow the same trend.

5. CONCLUSION

From the result obtained from this work, it was observed that the separation between the geoid and the reference ellipsoid in Taraba State University Jalingo, are all positive. The Ellipsoidal height was observed from Differential Global Positioning System (DGPS) data ie (X, Y, Z), the orthometric height was computed from the two known height with Excel using the formula $N = H - h$.

Base on the calculated root mean square, the height difference between the Geoid and the reference Ellipsoid indicates that there is a significant and adequate distance between the Geoid and the reference Ellipsoid. From Figure 4, it further indicates that the Geoid is an equipotential surface and Ellipsoidal and Orthometric heights also follow the same pattern. The trends of the plots of Ellipsoidal and Orthometric heights in chart and contour (figure 4 and 5) show how the patterns of the heights are the same. This is an indication that the two height systems are true representation of the same terrain.



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